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Higgs-Mediated Neutrinoless Double β -Decay and Neutrino Mass in a Majoron Model

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ABSTRACT

We propose a simple extension of the standard model in which the neutrinoless double β -decay is a tree level effect mediated by Higgs bosons and the neutrino mass begins to appear in the two-loop diagrams. The model also contains a massless majoron, *i.e.* the Goldstone boson of the global symmetry of the lepton number, which is spontaneously broken at a scale close to that of the electroweak interaction. No unnaturally huge or tiny new scales are evoked. The majoron coupling to the fermions is also one-loop suppressed.

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Introduction. The relationship between the neutrinoless double β -decay, $(\beta\beta)_{0\nu}$, and the neutrino mass has been a subject of constant theoretical interest. The questions can be posed at two different levels. First of all, one can ask, if the electron neutrino is massless or of massive-Dirac type, can $(\beta\beta)_{0\nu}$ occur? Or, the converse question, if $(\beta\beta)_{0\nu}$ occurs, can the electron neutrino avoid a Majorana mass? The answers to both questions are negative.¹ That is, the existence of $(\beta\beta)_{0\nu}$ implies in principle that the electron neutrino has non-zero Majorana mass, and vice versa. This is because given a six fermion vertex responsible for the non-vanishing $(\beta\beta)_{0\nu}$, one can construct a four-loop diagram that, barring from accidental cancellation, will give rise to non-zero Majorana mass for the electron neutrino. On the other hand, given a non-zero Majorana electron neutrino mass, a non-vanishing $(\beta\beta)_{0\nu}$ will arise through the diagram in Fig. 1 which we shall call the “*standard*” contribution.

Given this situation, one then asks whether it is possible to construct a model in which the standard contribution is *not* the main source to the $(\beta\beta)_{0\nu}$. Many attempts have been made. However all the discussions in the literature so far involve either a new scale² or new fermions³ or both. In this paper, we like to demonstrate that by a simple extension of the Higgs sector of the standard model one can arrive at an elegant model in which the $(\beta\beta)_{0\nu}$ is mainly due to a Higgs-mediated diagram. This is⁴ because the neutrinos are massless at the tree level and begin to pick up non-zero masses at the two-loop level while the Higgs-mediated $(\beta\beta)_{0\nu}$ is a tree level effect. The simplest model contains a global lepton number symmetry which is spontaneously broken near the electroweak scale. No new scale or new fermions are required in the model. A massless Goldstone boson, the majoron J , exists as the remnant of the original lepton number symmetry.

Majoron couplings to the fermions are one-loop suppressed. It is also interesting to discuss what happens if one does not impose the lepton number symmetry. In that case, the tree level Higgs-mediated $(\beta\beta)_{0\nu}$ and the one-loop level neutrino mass involve completely independent lepton number breaking parameters.

Model. For simplicity, our discussion will use only one generation of fermions in the $SU(2)_L \times U(1)_Y$ model unless otherwise specified. In the Higgs sector, in addition to the usual doublet ϕ , we add two singlets, h_+ and k_{++} , and one complex triplet T with zero hypercharge ($Y = 0$). The model is a hybrid combination of two models appeared in Ref. 5. Note that the triplet we have used has different hypercharge compared with the triplet that occurs so often in the discussion^{1,2} of $(\beta\beta)_{0\nu}$. The definition of the majoron model is completed by imposing the lepton number L :

$$\begin{aligned} L(\phi) &= 0, & L(\psi_L) &= L(e_R) = 1, \\ L(k_{++}) &= -2, & L(h_+) &= L(T) = -1. \end{aligned}$$

Here $\psi_L = (\nu, e)_L^T$. With this lepton number symmetry, the allowed Yukawa couplings are:

$$\mathcal{L}_Y = (\sqrt{2}m_e/v_2)\phi\bar{\psi}_L e_R + g_{ee}k_{++}e_R^T C e_R + h.c. , \quad (1)$$

and some of the crucial scalar self-couplings are

$$\mathcal{L}_H = \lambda\phi^T \bar{\tau}\phi \cdot \vec{T}h^{++} + Mh_+h_+k_{++}^* + h.c. \quad (2)$$

For later discussion, other interesting couplings that are forbidden by the sym-

metry of the lepton number are

$$\mathcal{L}_X = \mu^2 T^2 + \alpha h_+ \psi_L^T C \psi_L + \beta \phi^\dagger \vec{\tau} \phi \cdot \vec{T} + h.c. \quad (3)$$

Note that the lepton number of h_+ is different from that in the models of Refs. 6 and 7. It is defined so that the crucial trilinear coupling $h_+ h_+ k_{++}^*$ is present. The $SU(2)_L \times U(1)_Y$ gauge symmetry and the lepton number global symmetry are spontaneously broken by $\langle \phi \rangle = v_2/\sqrt{2}$ and $\langle T \rangle = v_3$. The experimental uncertainty in ρ parameter translates⁵ into the upper bound $(v_3/v_2) \leq 0.04$. In all the subsequent numerical estimate, we will advocate

$$v_3 \simeq 0.04 v_2, \quad (4)$$

so that the necessity of a new unnatural scale is avoided.

The unphysical neutral Higgs, which is absorbed by the Z boson, is purely $Im(\phi_0)$ because the neutral component T_0 of the triplet T does *not* carry I_{3L} charge. More crucially, the unphysical charged Higgs ω_+ (to be absorbed by W^+) is a linear combination of ϕ_+, T_+ and T_-^* ,

$$\omega_+ = (v_2^2 + 8v_3^2)^{-\frac{1}{2}} (v_2 \phi_+ + 2v_3 T_+ - 2v_3 T_-^*). \quad (5)$$

Due to the quartic coupling λ in Eq. (2), the vacuum expectation values (v.e.v.) induce a mixing between ϕ_+ and h_+ , and this effect is essential for the neutrino mass generation and the neutrinoless double β -decay. The physical charged Higgs bosons ($H_i^+, i = 1, 2, 3$) are linear superpositions of ϕ_+, T_+, T_-^* and h_+ . The masses of H_i^+ are obtained by the diagonalization of the mass matrix from the Higgs potential. We notice that the tachyon problem exists in the charged

Higgs sector. However, the tachyon problem can be easily cured by adding a gauge singlet s with global lepton number in this model. This is achieved by the additional interactions $s^2 T^2$ and $\phi^\dagger \vec{\tau} \phi \cdot \vec{T} s$ for $L(s) = 1$ as shown in the appendix. The majoron is then composed of only two pieces $Im T_0$ and $Im s$ to all orders in perturbation.

Neutrino mass. The leading contribution to the neutrino mass begins at the two-loop level as shown in Fig. 2. The mixing between ϕ_+ and h_+ plays an important role here. The unphysical Goldstone boson ω_+ cannot contribute because of the absence of the h_+ component in Eq. (5). From the unitarity condition and Eq. (5), the physical charged Higgs boson H_i^+ contains a small component of ϕ^+ about the size $\sqrt{2}v_3/v_2$. The ultraviolet divergence of the diagram in Fig. 2 is absent for the reason of the GIM mechanism⁸ in the charged Higgs sector. If the charged Higgs bosons H_i^+ have a degenerate mass, their finite contributions will also exactly cancel each other. Based on dimensional counting, the electron neutrino mass is

$$m_\nu = D \left(\frac{1}{16\pi^2} \right)^2 \left(\frac{\sqrt{2}v_3}{v_2} \right)^2 \left(\frac{\sqrt{2}m_e}{v_2} \right)^2 g_{ee} M, \quad (6)$$

where D denotes the uncertainty in this estimate and is expected to be of order one when the masses of H_i^+ are not degenerate and they are of the same order of magnitude as m_k . This gives

$$m_\nu = (2.7 \times 10^{-7} \text{ eV}) D (g_{ee} M / 250 \text{ GeV}),$$

which is far smaller than the bound $m_\nu < 1 \text{ eV}$ from the $(\beta\beta)_{0\nu}$ data⁹ of

$^{76}\text{Ge} \rightarrow ^{76}\text{Se}.$

Neutrinosless double β -decay. The leading tree-level mechanism for the $(\beta\beta)_{0\nu}$ is demonstrated in Fig. 3. It also involves the mixing between ϕ_+ and h_+ which is moderately suppressed by $(\sqrt{2}v_3/v_2)$. The effective Hamiltonian for $(\beta\beta)_{0\nu}$ is

$$\mathcal{H}_{\text{eff}} = G_2 (\bar{d}_L u_R)^2 e_R^T C e_R + h.c. , \quad (7)$$

with

$$G_2 = \left(\frac{\sqrt{2}m_q}{v_2} \right)^2 \left(\frac{g_{ee}M}{m_k^2} \right) \left(\sum_i \frac{U_{\phi i} U_{ih}^\dagger}{m_{H_i^+}^2} \right)^2 . \quad (8)$$

Knowing that the mixing amplitude $U_{\phi i}$ is about the size $\sqrt{2}v_3/v_2$, we estimate the last term in the above equation by the generic mass scale m_+ of the singly charged Higgs bosons, *i.e.*

$$\left(\frac{1}{m_+^2} \right) \left(\frac{\sqrt{2}v_3}{v_2} \right) \simeq \left(\sum_i \frac{U_{\phi i} U_{ih}^\dagger}{m_{H_i^+}^2} \right) . \quad (9)$$

The lifetime $\tau(\beta\beta)_{0\nu}$ of $(\beta\beta)_{0\nu}$ is proportional to the square of

$$(G_2 m_p / G_F^2) \langle f | \Omega_\Delta | i \rangle ,$$

with the dimensionless nucleus matrix element $\langle f | \Omega_\Delta | i \rangle$ defined in Ref. 2. Note that G_2 is very sensitive to the charged Higgs masses from the propagators while the neutrino mass is not as the dependence is tamed by the loop integrations for $m_k \simeq m_+$. To see how large the Higgs contribution can be, we have to look for the experimental constraints on m_{H^+} and m_k . In the lowest order, the singly charged Higgs boson H_i^+ only couples to fermions through the mixing with ϕ_+ .

Therefore, there is no strong constraint on its mass other than that from the e^+e^-

collider experiment at PETRA which gives $m_{H_i^+} \geq 20\text{GeV}$. The constraints on m_k and its coupling g_{ee} have been analyzed by Babu.⁶ If one assumes that the flavor changing coupling k_{++} is very small, then the value of 20 GeV can also be taken as the lower bound for m_k . Therefore the amplitude ratio of the Higgs contribution to the standard contribution for $(\beta\beta)_{0\nu}$ is of the order

$$\begin{aligned} & (16\pi^2)^2 \left(\frac{m_q}{m_e}\right)^2 \frac{m_e m_p}{m_k^2 m_+^4 G_F^2} \frac{\langle f|\Omega_\Delta|i\rangle}{\langle f|\Omega_\nu|i\rangle} \\ & \simeq 2 \times 10^5 \frac{\langle f|\Omega_\Delta|i\rangle}{\langle f|\Omega_\nu|i\rangle} \gg 1, \end{aligned} \quad (10)$$

with the current quark mass $m_q \simeq 15$ MeV, the charged Higgs generic mass $m_+ \simeq 20$ GeV and $m_k \simeq 50$ GeV. For a lengthy review of the nucleus matrix element, see Refs. 10, 11 and 12. Most of the attention in the literature has been paid to the nuclear matrix element $4\pi \langle f|\Omega_\nu|i\rangle \simeq 1$ for the standard neutrino contribution. To estimate the Higgs contribution to $(\beta\beta)_{0\nu}$, the most serious uncertainty comes from the nuclear matrix element $\langle f|\Omega_\Delta|i\rangle$. It has been studied only for the process $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$. In different nuclear models and calculations, it varies,

$$\begin{aligned} 4\pi \langle f|\Omega_\Delta|i\rangle & \simeq 284 \quad (\text{Ref. 12}), \\ & \simeq 106 \sim 263 \quad (\text{Ref. 13}). \end{aligned} \quad (11)$$

The experimental bound for the process $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$ is translated^{2,12} into

$$|(G_2 m_p / G_F^2) 4\pi \langle f|\Omega_\Delta|i\rangle|^2 \leq 0.62 \times 10^{-8}.$$

In this model, it becomes

$$4\pi \langle f|\Omega_\Delta|i\rangle \left(\frac{g_{ee} M}{250\text{GeV}}\right) \left(\frac{50\text{GeV}}{m_k}\right)^2 \left(\frac{20\text{GeV}}{m_+}\right)^4 \leq 790. \quad (12)$$

Therefore the Higgs contribution can be close to the experimental upper bound

for $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$. It would be interesting to obtain a better bound if the nuclear matrix element $\langle f|\Omega_\Delta|i\rangle$ for the process $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ is available.

Majoron. One may worry that breaking the lepton number symmetry at such a moderate v.e.v. v_3 will conflict with the known astrophysical constraint like the stellar energy loss due to the majoron emission.¹⁴ That is not true because the majoron coupling to fermions are suppressed by the one-loop effect just like that in the models of Ref. 5. A typical diagram for such coupling is given in Fig. 4. This part of the phenomenology is not much different from those in Ref. 5. The ordinary neutral physical Higgs boson H , is invisible¹⁵ in this model, because it can decay mainly into two majoron JJ .

Neutrino oscillation. If the Yukawa couplings of k_{++} to the leptons are flavor diagonal, then the neutrino mass matrix will be diagonal also. This feature is distinguished from that of the Babu's model⁶. Since we have assumed the Yukawa couplings of k_{++} are almost diagonal in order to allow a low mass value for k_{++} , the neutrino oscillation will be negligible. Therefore, observations of both the neutrino oscillation and the $(\beta\beta)_{0\nu}$ will pose a serious problem for this model.

Explicit breaking. If we remove the requirement of the lepton number symmetry in the Lagrangian, the previously forbidden terms in Eq. (3) will be present. We can further make the model more economical by using the *real* triplet instead of the *complex* one. The α term in Eq. (3) gives an one-loop Majorana neutrino mass matrix, which is off-diagonal and antisymmetric in the manner of the Zee's model.¹⁶ The Higgs-mediated $(\beta\beta)_{0\nu}$ occurs, as before, at the tree level. Therefore m_ν and $(\beta\beta)_{0\nu}$ involve different lepton number breaking parameters. The presence of the μ term in Eq. (3) can evade the tachyon problem even without introducing a gauge singlet s as in the above study of the H_i^+ masses.

Conclusion. We have shown a simple majoron model in which the $(\beta\beta)_{0\nu}$ is mainly mediated by the Higgs exchange at the tree level and the neutrino mass begins to appear at the two-loop diagram. The model does not require any new and unnatural scale other than that of the electroweak interaction.

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Appendix

We will analyze the Higgs potential of the model and show that the vacuum we used is indeed a minimum. The Higgs potential can be written as

$$\begin{aligned} \mathcal{H}(\phi, T, h_+, k_{++}, s) = & -\mu_0^2 s^* s + \lambda_0 (s^* s)^2 + \mu_1^2 h^* h + \lambda_1 (h^* h)^2 \\ & - \mu_2^2 \phi^\dagger \phi + \lambda_2 (\phi^\dagger \phi)^2 - \mu_3^2 T^\dagger T + \lambda_3 (T^\dagger T)^2 + \lambda'_3 (T^\dagger \bar{t} T)^2 \\ & + \lambda_{12} h^* h \phi^\dagger \phi + \lambda_{13} h^* h T^\dagger T + \lambda_{23} T^\dagger T \phi^\dagger \phi + \lambda'_{23} T^\dagger \bar{t} T \cdot \phi^\dagger \bar{\tau} \phi \\ & + [\lambda \phi^T \bar{\tau} \phi \cdot \bar{T} h^* + \beta_1 T^2 s^2 + \sqrt{2} \beta_2 \phi^\dagger \bar{\tau} \phi \cdot \bar{T} s + \text{h.c.}] + V(k) , \end{aligned}$$

where $V(k)$ includes all the terms involving at least one k_{++} . Since $V(k)$ does not affect our analysis of the stability of the vacuum, we do not need it explicitly.

The vacuum expectation values are $\langle s \rangle = v_0$, $\langle \phi \rangle = v_2/\sqrt{2}$ and $\langle T \rangle = v_3$.

$$\begin{aligned} (-\mu_0^2 + 2\lambda_0 v_0^2 + 2\beta_1 v_3^2) 2v_0 - \beta_2 v_3 v_2^2 &= 0 & (\text{for } v_0) \\ -\mu_2^2 + \lambda_2 v_2^2 + \lambda_{23} v_3^2 - 2\beta_2 v_3 v_0 &= 0 & (\text{for } v_2) \\ (-\mu_3^2 + 2\lambda_3 v_3^2 + \lambda_{23} v_2^2/2 + 2\beta_1 v_0^2) 2v_3 - \beta_2 v_0 v_2^2 &= 0 & (\text{for } v_3) . \end{aligned}$$

Using these relations, the singly charged mass-squared matrix for the ordered

basis of ϕ_+ , h_+ , T_+ and T_-^* can be written as

$$\begin{pmatrix} 16b_2u_0u_3 & -4lu_3 & -2u_3 - 4b_2u_0 & -2u_3 + 4b_2u_0 \\ a & 2l & 0 & \\ & 1+4l'u_3^2-8b_1u_0^2+2b_2\frac{u_0}{u_3} & -4l'u_3^2-8b_1u_0^2 & \\ & & -1+4l'u_3^2-8b_1u_0^2+2b_2\frac{u_0}{u_3} & \end{pmatrix}.$$

Note that T_+ and T_-^* are independent complex fields. Here a common factor $M^2 = \lambda'_{23}v_2^2/4$ has been taken away and $u_{0,3} = v_{0,3}/v_2$, $l = \lambda/\lambda'_{23}$, $l' = \lambda'/\lambda'_{23}$, $b_1 = \beta_1/\lambda'_{23}$, $b_2 = \beta_2/\lambda'_{23}$ and $a = (\mu_1^2 + \lambda_{12}v_2^2/2 + \lambda_{13}v_3^2)/M^2$. The mass matrix is of course symmetric. We have assumed all the couplings to be real. It is easy to check that the Goldstone boson is proportional to the vector $(1, 0, 2u_3, -2u_3)$ which has zero eigenvalue. When the singlet is not introduced, then effectively $b_1 = b_2 = 0$, and the 2×2 block of indices T_+ and T_-^* has a negative determinant. This implies that the matrix has at least one negative eigenvalue and the vacuum is unstable.

When the singlet is included, it is easy to see that this technical difficulty is avoided by taking some special, but resonable, limit for some of the parameters. For $u_3 \ll u_0$ and $8b_1 \simeq 2b_2 \simeq 1$, the mass matrix reduces to

$$\begin{pmatrix} 0 & 0 & -4b_2u_0 & 4b_2u_0 \\ a & 0 & 0 & \\ & -8b_1u_0^2 + 2b_2u_0/u_3 & -8b_1u_0^2 & \\ & & -8b_1u_0^2 + 2b_2u_0/u_3 & \end{pmatrix}.$$

To a good approximation the nonzero eigenvalue are a and $2b_2u_0/u_3$ with double degeneracy. Any off diagonal elements can be treated as perturbation to these values. Therefore the vacuum is a true minimum. However, $u_3 \ll u_0$ is in no way a necessary condition. Solutions with $u_3 \simeq u_0$ may also exist.

Note that in order to enhance the amplitude of $(\beta\beta)_{0\nu}$, light charged Higgs bosons H_i^+ are preferred in Eq. (8). Since the coupling g_{eeJ} of the Majoron J to the electron is only one-loop suppressed as shown in Fig. 4, a light H_i^+ presents a potential problem with the astrophysical bound $g_{eeJ} \leq 10^{-12}$. It turns out that the introduction of the singlet s also allows us to avoid this problem. If we assume $v_0 \gg v_3$, then the one-loop diagram in Fig. 4 will be further suppressed by a factor of v_3/v_0 . Therefore, a light $M_{H_i^+}$ can be easily accommodated. However, this is not the full story because $Im\ s$ can also have one-loop and off-shell mixing with $Im\ \phi_0$ through the coupling β_2 in the Higgs potential \mathcal{H} . This results in the derivative coupling of the Majoron to the electron. Assuming β_2 is real, such a term will give rise to couplings:

$$\begin{aligned} & Re\ s\ Re\ \phi_0\ \phi_+(T_- - T_+^*) + Im\ s\ Im\ \phi_0\ \phi_+(T_- + T_+^*) \\ & -i\ Re\ s\ Im\ \phi_0\ \phi_+(T_- - T_+^*) + iIm\ s\ Re\ \phi_0\ \phi_+(T_- + T_+^*) + h.c. \end{aligned}$$

Since $T_- \pm T_+^*$ are orthogonal combinations, if they do not mix with each other, the one-loop and off-shell mixing between $Im\ s$ and $Im\ \phi_0$ will be zero. The dominant contribution to their mixing is provided by the β_1 coupling. By this observation, the one-loop coupling of $Im\ s$ to the electron is estimated to be

$$\frac{\beta_2^2 \beta_1}{16\pi^2} \frac{m_e v_0^3}{M_T^4},$$

where the mass scale M_T^2 of T_\pm is estimated to be $\beta_2 v_2^2 v_0 / v_3$. Therefore the coupling of J to the electron due to $Im\ s$ is about

$$\begin{aligned} & \frac{\beta_1}{16\pi^2} \left(\frac{m_e}{v_2}\right) \left(\frac{v_3}{v_2}\right)^2 \left(\frac{v_0}{v_2}\right) \\ & \sim 10^{-2} \cdot 10^{-2} \cdot 10^{-6} \cdot 10^{-3} \cdot 1 \sim 10^{-13}. \end{aligned}$$

While the contribution due to $Im\ T$ in Fig. 4 is

$$\frac{\lambda^2}{16\pi^2} \left(\frac{m_e v_3}{m_+^2} \right) \left(\frac{v_3}{v_0} \right) \sim 10^{-13} ,$$

for $v_3 \sim 10\text{ GeV}$, $m_+ \sim 20\text{ GeV}$, $\lambda \sim \beta_1 \sim 10^{-2}$. They both satisfy the astrophysical bound of 10^{-12} .

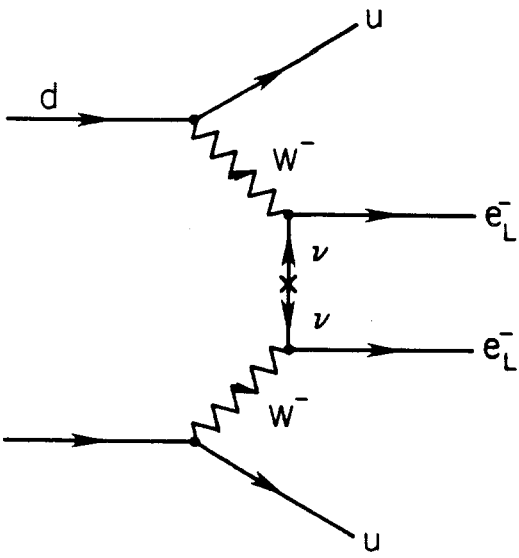
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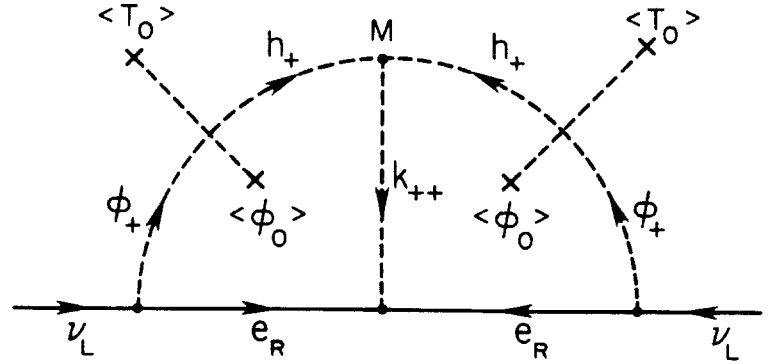
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FIGURE CAPTIONS

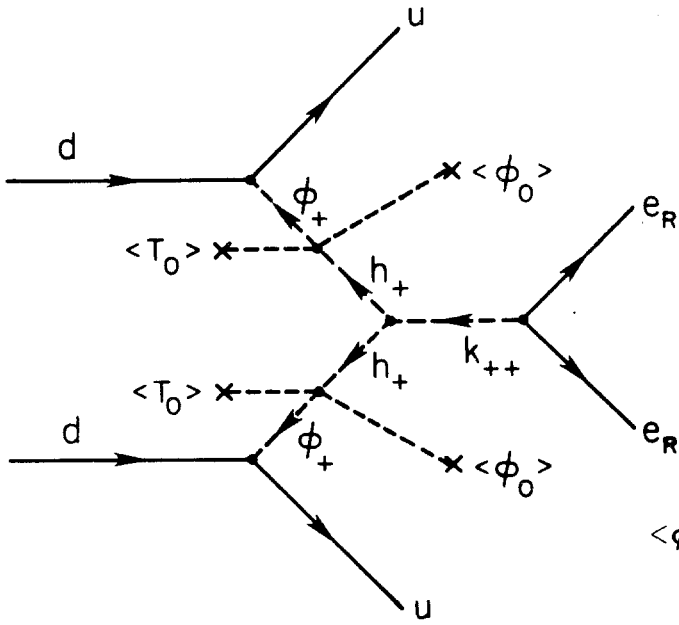
1. The “standard contribution” to $(\beta\beta)_{0\nu}$ through the neutrino mass.
2. The two-loop diagram for the Majorana neutrino mass.
3. The diagram for the $(\beta\beta)_{0\nu}$ mediated by the charged Higgs bosons.
4. A typical one-loop diagram for the induced majoron coupling to the electron.



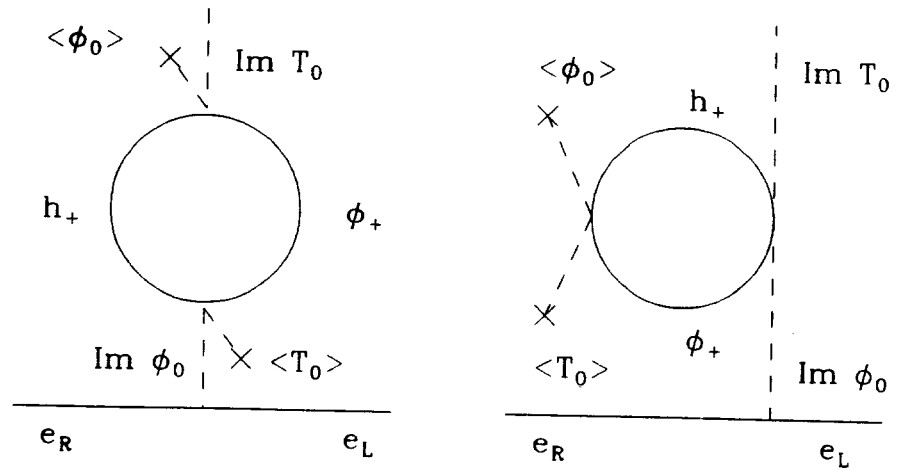
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